nefin

FEA – USP

Methodology

Index

1 Introduction

The objective of this document is to explain the methodology used in the construction of the variables that are available for download in our website (http://www.fea.usp.br/nefin/).

2 Portfolios

Portfolios are constructed by sorting "eligible" (eligibility is explained is Section 2) BOVESPA stocks according to size, book-to-market, momentum, illiquidity and industry sector. We compute value and equal-weighted returns of the following portfolios:

- 3 portfolios sorted by size;
- 3 portfolios sorted by book-to-market;
- 3 portfolios sorted by momentum;
- 3 portfolios sorted by illiquidity;
- 4 portfolios sorted by size and by book-to-market (2x2);
- 4 portfolios sorted by momentum and by size (2x2);
- 4 portfolios sorted by size and illiquidity (2x2);
- 7 portfolios sorted by industry;

Each group of portfolios is available for download in Excel files. In each file you will find worksheets with value-weighted returns, equal-weighted returns, number of stocks, average market value, average book value and average book-to-market ratio. How we constructed these variables is explained below.

2.1 Value-Weighted Returns (daily)

The *Value-weighted Returns* of portfolio *P* in day *t* is computed as

$$\boldsymbol{R_t} = \sum_{\boldsymbol{i}} \boldsymbol{\omega_{i,t} r_{i,t}}$$

Where:

- $\omega_{i,t}$ is the weight of stock *i* on day *t*. It is the ratio between the *t-1* market value of stock *i* and *t-1* total market value of *P*;

- $r_{i,t}$ is the return of stock *i* on day *t*, which is computed as

$$r_{i,t} = \begin{cases} 0 & \text{if there is no trade of i in t,} \\ \dfrac{P_t^i}{P_{t-1}^i} - 1 & \text{otherwise;} \end{cases}$$

- P_t^i is the price of stock *i* on day *t* adjusted for dividends and splits.

2.2 Equal-weighted Returns (daily)

The *Equal-weighted Returns* of portfolio *P* on day *t* is computed as

$$\boldsymbol{R_t} = \frac{1}{N}\sum_{\boldsymbol{i}} \boldsymbol{r_{i,t}}$$

Where:

- *N* is the number of stocks in the portfolio;

- $r_{i,t}$ is the return of stock *i* on day *t* defined in Section 1.1.

2.3 Number of Stocks (monthly)

The *Number of Stocks* in portfolio *P* in month *t* is the total number of stocks that belong to portfolio *P*.

2.4 Average Market Value (monthly)

The *Average Market Value* of a given portfolio in month *t* is the simple average of the market values, in thousands of reais, of the stocks in the portfolio (the market value of a stock is the market value of the firm the stock belongs to).

2.5 Average Book Value (annual)

The *Average Book Value* of a given portfolio in year *t* is the simple average of the book values, in thousands of reais, of the stocks in the portfolio (the book value of a stock is the book value of the firm the stock belongs to). We use book values of June.

2.6 Average Book-to-market (annual)

The *Average Book-to-market* of a given portfolio in year *t* is the simple average of the book-to-market ratio of the stocks in the portfolio (the book-to-market of a stock is the book-to-market of the firm the stock belongs to). We use book-to-market ratios of June.

3 Eligibility criteria

A stock traded in BOVESPA is considered "eligible" for year *t* if it meets 3 criteria:

- The stock is the most traded stock of the firm (the one with the highest traded volume during last year);
- The stock was traded in more than 80% of the days in year *t-1* with volume greater than R$ 500.000,00 per day. In case the stock was listed in year *t-1,* the period considered goes from the listing day to the last day of the year;
- The stock was initially listed prior to December of year *t-1*.

4 Portfolios methodology

4.1 3 portfolios sorted by size

Every January of year *t*, we (ascending) sort the eligible stocks (as defined in Section 2) in terciles according to their market capitalization in December of year *t-1* (the market capitalization of a stock is the market capitalization of the firm the stock belongs to). We then hold the portfolios during year *t*.

4.2 3 portfolios sorted by book-to-market

Every January of year *t*, we (ascending) sort the eligible stocks (as defined in Section 2) in terciles according to their book-to-market ratio in June of year *t-1* (the book-to-market ratio of a stock is the book-to-market ratio of the firm the stock belongs to). We then hold the portfolios during year *t*.

4.3 3 portfolios sorted by momentum

Every month *t*, we (ascending) sort the eligible stocks (as defined in Section 2) in terciles according to their cumulative returns from month *t-12* and month *t-2*. We then hold the portfolios during month *t*.

4.4 3 portfolios sorted by illiquidity

Every month t, we (ascending) sort the eligible stocks (as defined in Section 2) in terciles according to their previous twelve month illiquidity moving average (stock illiquidity is computed as in Acharya and Pedersen 2005). We then hold the portfolios during month t.

4.5 4 portfolios sorted by size and by book-to-market (2x2)

Every January, we double-sort (ascending) the eligible stocks (as defined in Section 2) according to 3.1 and 3.2. We then hold the portfolios during year *t*.

4.6 4 portfolios sorted by size and by momentum (2x2)

Every month, we double-sort (ascending) the eligible stocks (as defined in Section 2) according to 3.1 (sorting the stocks by size every month) and 3.3. We then hold the portfolios during month *t*.

4.7 4 portfolios sorted by size and by illiquidity (2x2)

Every month, we double-sort (ascending) the eligible stocks (as defined in Section 2) according to 3.1 (sorting the stocks by size every month) and 3.4. We then hold the portfolios during month t.

4.8 7 portfolios sorted by industry

We classify the eligible stocks (as defined in Section 2) into the following industry sectors: Basic Products, Construction, Consumer, Energy, Finance, Manufacturing, and Other.

5 Risk Factors

5.1 Market Factor

The *Market Factor* is the difference between the value-weighted daily return of the market portfolio (using all the eligible stocks as defined in Section 2) and the daily risk-free rate. The daily risk-free rate is computed from the 30-day DI Swap.

5.2 Small Minus Big (SMB)

The *Small Minus Big Factor (SMB)* is the return of a portfolio long on stocks with low market capitalization ("Small") and short on stocks with high market capitalization ("Big").

Every January of year *t*, we (ascending) sort the eligible stocks according to their December of year *t-1* market capitalization, and separate them into 3 quantiles. Then, we compute the equal-weighted returns of the first portfolio ("Small") and the third portfolio ("Big"). The SMB Factor is the return of the "Small" portfolio minus the return of the "Big" portfolio.

5.3 High Minus Low (HML)

The *High Minus Low Factor (HML)* is the return of a portfolio long on stocks with high book-to-market ratio ("High") and short on stocks with low book-to-market ratio ("Low").

Every January of year *t*, we (ascending) sort the eligible stocks into 3 quantiles (portfolios) according to the book-to-market ratio of the firms in June of year *t-1*. Then, we compute the equal-weighted returns of the first portfolio ("Low") and the third portfolio ("High"). The HML Factor is the return of the "High" portfolio minus the return of the "Low" portfolio.

5.4 Winners Minus Losers (WML)

The *Winners Minus Losers Factor (WML)* is the return of a portfolio long on stocks with high past returns ("Winners") and short on firms with low past returns ("Losers").

Every month *t*, we (ascending) sort the eligible stocks into 3 quantiles (portfolios) according to their cumulative returns between month *t-12* and *t-2*. Then we compute the equal-weighted returns of the first portfolio ("Losers") and the third portfolio ("Winners"). The WML Factor is the return of the "Winners" portfolio minus the return of the "Losers" portfolio.

5.5 Illiquid Minus Liquid (IML)

The *Illiquid Minus Liquid Factor (IML)* is the return of a portfolio long on stocks with high illiquidity ("Illiquid") and short on stocks with low illiquidity ("Liquid").

Every month *t*, we (ascending) sort the eligible stocks into 3 quantiles (portfolios) according to their previous twelve month illiquidity moving average (stock illiquidity is computed as in Acharya and Pedersen 2005). Then we compute the equal-weighted returns of the first portfolio ("Liquid") and the third portfolio ("Illiquid"). The *IML* Factor is the return of the "Illiquid" portfolio minus the return of the "Liquid" portfolio.

6 Illiquidity Index

The *Illiquidity of stock i* is a measure of how its *stock* price moves in response to the its *traded volume*. We construct this measure as in Acharya and Pedersen (2005):

$$ILLIQ_t^i = \min\left\{ \frac{1}{Days_t^i} \sum_{d=1}^{Days_t^i} \frac{\left|r_{td}^i\right|}{V_{td}^i / P_{t-1}^M}, 30.00 \right\},$$

Where:

- $Days_t^i$ is the number of days in *month t* for *stock i* was traded*;*

- r_{td}^i is the return of stock *i* on day *d, month t,* defined in Section 1.1;

- V_{td}^i is the traded volume (in millions) of stock *i* on day *d, month t*;

- P_{t-1}^M is the ratio between market capitalizations of the market portfolio at the end of *month t-1* and at the end of January 2000.

The *Illiquidity Index is* the value weighted *illiquidity* of the whole market in Brazil: the value weighted average of the illiquidity of each eligible stock.

7 Cost of Capital

We compute the cost of capital for each sector contained in the "7 portfolios sorted by industry" according to CAPM methodology described below.

We run the regression of monthly excess returns for each industry on the monthly market risk factor, all data available at nefin. With the purpose to obtain the cost of capital for 1,5,10 and 20-year projects, we then multiply the resulting betas by monthly

US market risk premium (obtained in Shiller's website) and then add real risk-free rates.

7.1 US market risk premium

For each of the above-mentioned maturities we calculate a risk premium by accumulating annual US market excess returns according to the respective moving window.

7.2 Real risk-free rate

The one year risk-free rate is computed from the 360-day DI Swap, deflated by the expected inflation as measured by the IPCA index (data available at the Brazilian Central bank wesite).

For the other maturities we use spline interpolations of the NTN-B rate.

8 Spot Rate Curve

We provide a spot rate curve by interpolating the One-Day Interbank Deposit futures contract (known in Brazil as the DI rate). We use a flat-forward interpolation, in which we assume that the yield rate in the interim period between two settlement dates is constant. The formula for the interpolated yield rate for date t is:

$$r_{t,T} = \left[\left(1 + r_{t,T-1}\right)^{\frac{du_{T-1}}{252}} \times \left(\frac{\left(1 + r_{t,T+1}\right)^{\frac{du_{t,T+1}}{252}}}{\left(1 + r_{t,T-1}\right)^{\frac{du_{t,T-1}}{252}}} \right)^{\frac{du_{t,T} - du_{t,T-1}}{du_{t,T+1} - du_{t,T-1}}} \right]^{\frac{252}{du_T}} - 1$$

Where:

- t: current date
- T: interpolation period (in our case, we use the following periods: one month, two months, three months, six months, one year, three years and five years)
- $T-1$: settlement date immediately prior to T
- $T+1$: settlement date immediately after T
- du_T: business days until interpolation date T
- du_{T-1}: business days until settlement date $T-1$
- du_{T+1}: business days until settlement date $T+1$
- $r_{t,T-1}$: day-t yield rate for the futures contract with settlement date $T-1$
- $r_{t,T+1}$: day-t yield rate for the futures contract with settlement date $T+1$

All business days counts are made considering a 252-day long business year.

9 Volatility Index

We compute the IVol-Br, a daily volatility index for the Brazilian market based on the paper by Carr and Wu (2006)[1].

The IVol-Br is the 2-month[2] (42 business days) expected volatility of the BOVESPA index (IBOVESPA). It is computed as the weighted average of the near-term and next-term volatilities of options over the IBOVESPA spot. At a given date t, the near-term refers to the closest expiration to t of the options over IBOVESPA, while the next-term refers to the expiration date immediately following the near-term[3].

The formula for the near and next-term volatilities is the following:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) \; - \; \frac{j}{T}\left[\frac{F}{K_0} - 1\right]^2 \qquad \textbf{(1)}$$

Where:

- $\sigma = {}^{volatility}/_{100}$;
- T: time until expiration;
- F: forward index level of the IBOVESPA, equal to its daily settlement price;
- K_0: the closest strike to the forward index, F;
- K_i: strike of the i-th out-of-the-money option: a call if $K_i > K_0$, a put if $K_i < K_0$, and both if $K_i = K_0$;
- ΔK_i: interval between strikes: half of the difference between the strikes immediately above and below K_i:
  - $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$
- R: risk-free interest rate until expiration T, from the daily settlement price of the futures interbank (DI) rate;

[1] Readers can access the paper by Carr and Wu here.
[2] While the volatility index in Carr and Wu (2006) is the 1-month expected volatility, we are restricted to calculating a 2-month volatility index for Brazil because the options over IBOVESPA only expire on even-numbered months.
[3] For instance, at any date on January 2015, the near-term refers to the options expiration date on February 2015, while the next-term refers to the expiration date on April 2015.

- $Q(K_i)$: market price of option K_i.

- j: adjustment factor ($j = 0{,}1\ or\ 2$), according to the following rule:

	$\boldsymbol{K_0 < F}$	$\boldsymbol{K_0 > F}$
$\exists\ call, \exists\ put$	j=1	j=1
$\exists\ call,\ \nexists\ put$	j=2	j=0
$\nexists\ call, \exists\ put$	j=0	j=2

  This adjustment is necessary in order to transform a in-the-money call (put) into its counterpart out-of-the-money put (call).

After calculating both the near-term and next-term volatilities, we then aggregate these into a weighted average which corresponds to the IVol-Br published at the NEFIN website. The formula for this aggregation is:

$$\text{Ivol-Br} = 100 \times \sqrt{\left\{T_1\sigma_1^2\left[\frac{N_{T_2}-N_{42}}{N_{T_2}-N_{T_1}}\right] + T_2\sigma_2^2\left[\frac{N_{42}-N_{T_1}}{N_{T_2}-N_{T_1}}\right]\right\} \times \frac{N_{252}}{N_{42}}} \quad \textbf{(2)}$$

- N_{T_1}: minutes until the near-term expiration date;

- N_{T_2}: minutes until the next-term expiration date;

- N_{42}: number of minutes in 42 business days (42 X 1440)

- N_{252}: number of minutes in 1 business year (252 X 1440)

It is important to note that this formula becomes an extrapolation in certain situations, when the weight of the next-term is negative (this happens right after the expiration of the near-term options, when the amount of days until the new expiration date for the near-term options is larger than 42 business days).

We also perform more adjustments to our methodology in order to adapt our volatility index to particular features of the Brazilian market:

- We restrict the set of options that is used to calculate the IVol-Br index to those traded between 3 p.m. and 6 p.m.;
- We use only the last trade that took place in the above-mentioned time interval for each ticker;
- We only calculate the near-term volatility index if there are at least 2 trades involving call options at different strikes and 2 trades involving put options also

at different strikes. This same condition applies to the calculation of the next-term volatility. This is done in order to circumvent errors associated with lack of liquidity in the options market;

- If in a given day the near-term volatility cannot be calculated, the IVol-Br index will be equal to the next-term volatility and vice-versa if the next-term volatility is unavailable but the near-term is. If both near and next-term volatilities cannot be calculated, we report the index for that day as missing;
- In days when the weight of the second term of equation **(2)** is negative, we ignore the next-term volatility, thus the IVol-Br index equals the near-term volatility.

Updated on January 28 2015

Questions or comments should be sent to nefin@usp.br.